ACHARYA NAGARJUNA UNIVERSITY

CURRICULUM - B.A / B.Sc

MATHEMATICS - PAPER - IV (ELECTIVE - 2)

90 hrs
(3hrs / week)

MODERN APPLIED ALGEBRA

UNIT - 1 (30 Hours)

1. SETS AND FUNCTIONS :

Sets and Subsets, Boolean algebra of sets, Functions, Inverses, Functions on S to S, Sums, Products and Powers, Peano axioms and Finite induction.

2. BINARY RELATIONS :

Introduction, Relation Matrices, Algebra of relations, Partial orderings, Equivalence relations and Partitions, Modular numbers, Morphisms, Cyclic unary algebras.

UNIT - 2 (20 Hours)

3. GRAPH THEORY :

Introduction - Definition of a Graph, Simple Graph, Konigsberg bridge problem, Utilities problem, Finite and Infinite graphs, Regular graph, Matrix representation of graphs - Adjacency matrix, Incidence matrix and examples; Paths and Circuits - Isomorphism, Sub graphs, Walk, Path, Circuit, Connected graph, Euler line and Euler graph; Operations on graphs - Union of two graphs, Intersecton of two graphs and ring sum of two graphs; Hamiltonian circuit, Hamiltonian path, Complete graph, Traveling salesmen problem. Trees and fundamental circuits, cutsets.

UNIT - 3 (25 Hours)

4. FINITE STATE MACHINES :

Introduction, Binary devices and states, Finite state machines, State diagrams and State tables of machines; Covering and Equivalence, Equivalent states, Minimization procedure.

5. PROGRAMMING LANGUAGES :

Introduction, Arithmetic expressions, Identifiers, Assignment statements, Arrays, For statements, Block strutures in ALGOL, The ALGOL grammar.

UNIT - 4 (15 Hours)

6. BOOLEAN ALGEBRAS :

Introduction, Order, Boolean polynomials, Block diagrams for gating networks, Connections with logic, Logical capabilities of ALGOL, Boolean applications.

Prescribed Text Book :

"Modern applied Algebra" by Dr. A. Anjaneyulu, Deepti publications, Tenali.

Reference Books :

1. Modern applied Algebra by Garrett Birkhoff and Thomas C.Bartee, CBS Publishers and Distributors, Delhi.
2. Graph Theory with applications to Engineering and Computer Science by Narsingh Deo, Prentice-Hall of India Pvt. Ltd., New Delhi.

ACHARYA NAGARJUNA UNIVERSITY

CURRICULUM - B.A / B.Sc

MATHEMATICS - PAPER - IV (ELECTIVE - 2)

MODERN APPLIED ALGEBRA

QUESTION BANK FOR PRACTICALS

UNIT - 1 (SETS AND FUNCTIONS, BINARY RELATIONS)

1. i) Is the cancellation law $A \cup B = A \cup C \Rightarrow B = C$ true? If not give example?

ii) Is the cancellation law $A \cap B = A \cap C \Rightarrow B = C$ true? If not give example?

2. Prove that $S \subset S \cap (S \cup T)$ and that $S \supset S \cap (S \cup T)$ and $S = S \cap (S \cup T)$.

3. If A, B, C are three sets, prove that $(A - B) - C = (A - C) - (B - C)$.

4. Find a necessary and sufficient condition for $S + T = S \cup T$ where $S + T = (S \cup T') \cap (S' \cup T)$.

5. Give an example of a function which is a surjection but not injection.

6. Show that the Peano's successor function is an injection but not surjection.

7. Find the number of functions from a finite set S of n elements to itself. Among these

i) how many are surjections ii) how many are injections?

8. Show that the functions $f(x) = x^3$ and $g(x) = x^{1/3}$ for $x \in R$ are inverses of one another.

9. Prove by induction in $P, m + r = m + s \Rightarrow r = s$.

10. If $f_1, f_2, \ldots\ldots, f_m$ are injections then show that $f_m \circ f_{m-1} \circ \ldots \circ f_1$ is an injection.

11. Prove by induction that $n^3 + 2n$ is divisible by 3 for all $n \geq 1$.

12. Prove by induction that $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ where n is any positive integer.

13. Let $X = \{a, b\}$ and $Y = \{c, d, e\}$. Write down the tabular representation for the relation α on X and Y defined by the list : $a\alpha\, c,\, a\alpha\, d,\, a\alpha'\, e,\, b\alpha'\, c,\, b\alpha'\, d,\, b\alpha\, e$.

14. Find the matrix of the relation α on $X = \{a, b\}$ and $Y = \{c, d, e\}$ which is defined by the list : $a\alpha\, c,\, a\alpha\, d,\, a\alpha'\, e,\, b\alpha'\, c,\, b\alpha'\, d,\, b\alpha\, e$.

15. Give an example of a relation which is neither reflexive nor irreflexive. Also give its graphical representation and relation matrix.

16. If ρ is symmetric, prove that $\rho \vee \rho^2 \vee \ldots \vee \rho^n$ is symmetric.

17. Show that a finite poset has a least element iff it has exactly one minimal element.

18. If ρ is reflexive and transitive then show that $\rho \wedge \overline{\rho}$ is an equivalence relation on a set S.

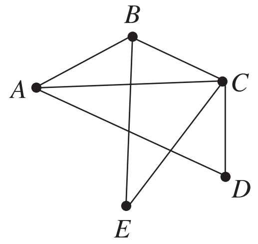
19. Let $A = (S, f)$ be a finite unary algebra with k elements. Define aRb in A to mean that for some $n \in N, f^n(a) = b$. Show that R is reflexive and transitive.

20. Let $A = [S, f]$ be any finite unary algebra with k elements. Define aRb in A to mean that for some $n \in N, f^n(a) = b$. Show that A is cyclic iff for some $a \in S$, aRb for all $b \in S$.

UNIT - 2 (GRAPH THEORY)

21. Draw the graph $G(V,E)$ where $V=\{A,B,C,D,E\}, E=\{AB,AC,BC,DE\}$.

22. Find the Edge set E of the graph $G=(V,E)$ given by



23. Explain Konigsberg bridge problem and draw its graph.

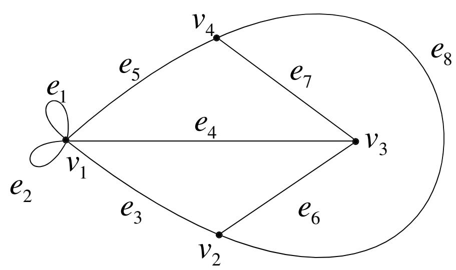
24. Explain three-utilities problem and draw its graph.

25. Draw a graph with the adjacency matrix $\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

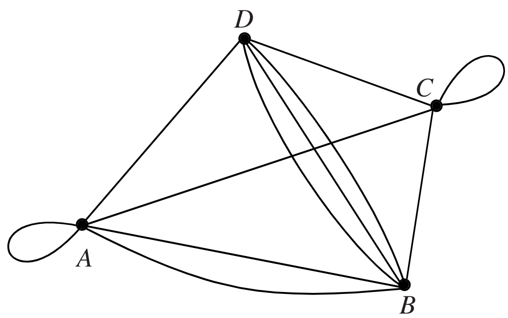
26. Draw a graph with the adjacency matrix $\begin{bmatrix} 2 & 1 & 3 & 0 \\ 1 & 0 & 1 & 2 \\ 3 & 1 & 0 & 1 \\ 0 & 2 & 1 & 1 \end{bmatrix}$

27. Draw a graph with the incidence matrix $\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$

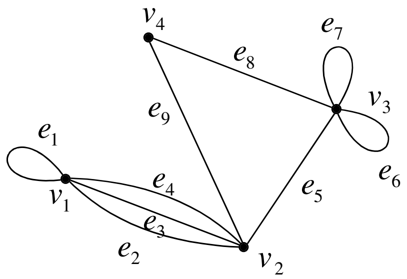
28. Find the adjacency matrix of the graph given by



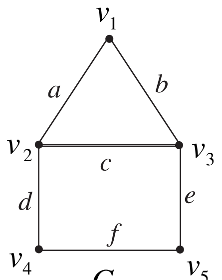
29. Find the adjacency matrix of the graph given by



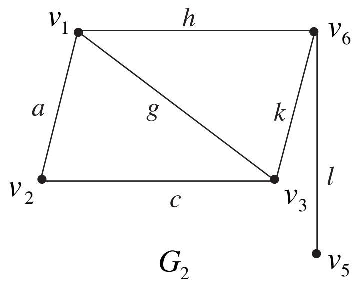
30. Find the incidence matrix of the graph given by



31. Find the union, intersection and ring sum of the following two graphs.

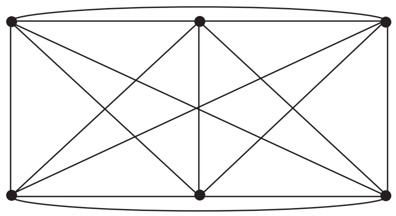


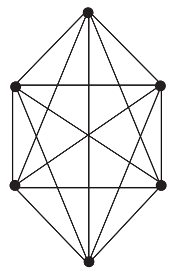
G_1



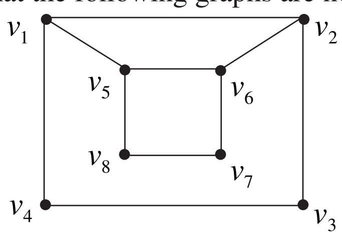
G_2

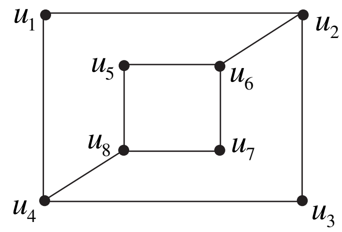
32. Show that the two graphs are isomorphic.



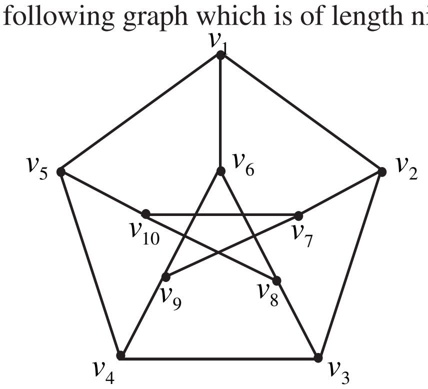


33. Show that the following graphs are not isomorphic to each other.

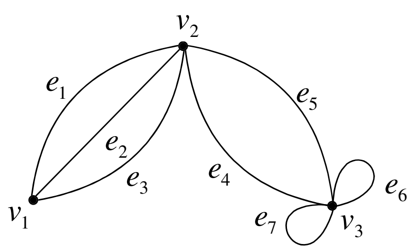




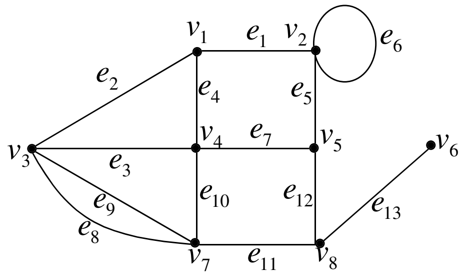
34. Draw a circuit from the following graph which is of length nine.



35. Draw all the circuits of the following graph.

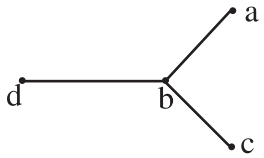


36. List all paths from v_1 to v_8 in the following graph.

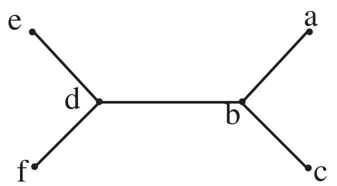


37. Find the eccentricity and centre of the graphs.

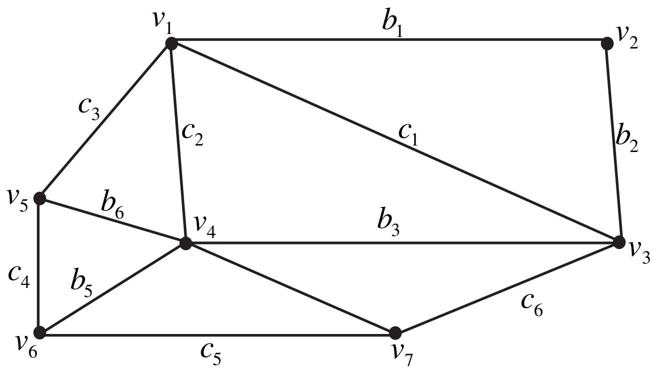
(i)



(ii)



38. Find the rank and nuclity of the spanning tree.



39. Draw all trees of three labeled vertices.

40. Find the edge connectivity of the complete graph of n -vertices.

UNIT - 3 (FINITE STATE MACHINES, PROGRAMMING LANGUAGES)

41. Find the output string when the input string 1101 is run into the following machine.

Present state	v		ξ	
	0	1	0	1
s_0	s_1	s_2	0	0
s_1	s_0	s_1	1	1
s_2	s_2	s_1	0	1

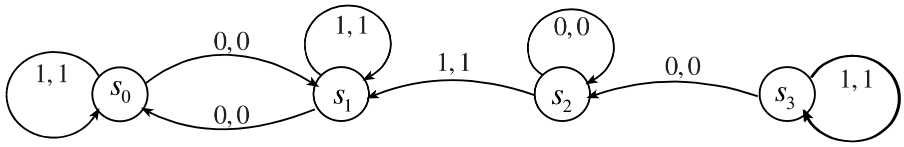
42. Give the state diagram of the following transition table.

Present state	Next state		Output	
	0	1	0	1
s_1	s_3	s_2	0	0
s_2	s_1	s_4	1	0
s_3	s_2	s_1	1	1
s_4	s_4	s_3	1	0

43. Draw the state diagram for the following machine

Present state	Next state 0	Next state 1	Output 0	Output 1
s_0	s_1	s_0	0	1
s_1	s_0	s_1	1	0
s_2	s_1	s_3	0	1
s_3	s_2	s_3	0	1
s_4	s_3	s_2	1	0
s_5	s_4	s_5	1	0

44. Write the state table of the following machine.



45. Establish a morphism from M and $\overline{M}$ given below :

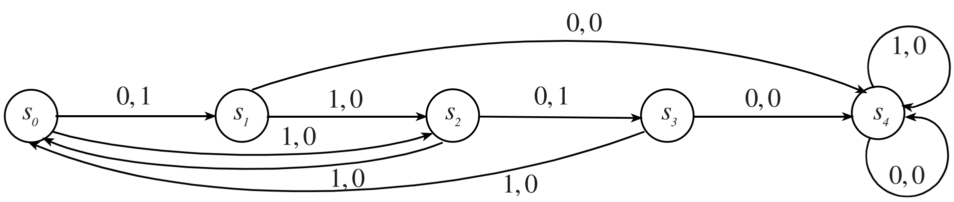
M	ν 0	ν 1	ξ 0	ξ 1
a	b	c	0	1
b	a	c	0	1
c	c	a	1	0

$\overline{M}$	$\overline{\nu}$ 0	$\overline{\nu}$ 1	$\overline{\xi}$ 0	$\overline{\xi}$ 1
1	1	2	0	1
2	2	1	1	0

46. Minimise the number of states in the following machine.

Present state	Next state a_0	Next state a_1	Output a_0	Output a_1
1	2	2	1	0
2	3	3	1	0
3	4	4	1	0
4	4	4	0	1
5	5	6	1	1
6	6	5	1	1

47. Minimise the number of states in the following machine.



48. Minimise the number of states in the following machine.

Present state	Next state a_0	Next state a_1	Output a_0	Output a_1
1	9	1	1	1
2	2	2	1	0
3	7	5	0	1
4	2	2	1	0
5	2	2	1	0
6	3	9	1	0
7	6	8	1	0
8	9	9	1	0
9	4	6	0	0

49. Write the ALGOL expressions for the following Mathematical expressions.

i) $a^3-b^3-3ab(a+b)$ ii) $AB+C^{D^E}$ iii) $\pi\ (r-p)^2$ iv) $P\left(1+\frac{r}{100}\right)^n$ v) $\log\left(\frac{x^2+y^2}{2xy}\right)$

50. Write the ALGOL expressions for the following Mathematical expressions.

i) $\sin^2 e^{x/y}$ ii) $\sin^3\left(x+\frac{y^2}{2}\right)$ iii) $\sqrt{e^A-\sin A^2}$ iv) $\frac{-b+\sqrt{b^2-4ac}}{2a}$ v) $\sqrt{1+\frac{1}{\sin|x|}}$

51. Convert the following ALGOL expressions to conventional mathematical expressions.

i) $A\times B-C$ ii) $A\uparrow(B-C)$ iii) $A\uparrow B-C$ iv) $A/B-C\uparrow D$ v) $A\div B-C\times D$

Evaluate the above for $A=2, B=3, C=4, D=5$.

52. Write the mathematical expression for the following ALGOL expressions

i) sqrt $(s\times(s-a)\times(s-b)\times(s-c))$ ii) sqrt $(\exp(\sin A)-\cos(A\uparrow 5))$

iii) $-b+\text{sqrt}(b\uparrow 2-4\times a\times c)/2\times a$ iv) $(\exp(x)\uparrow x+\exp(1/x)/(\exp(x\uparrow 2)+\text{sqrt}(x))$

v) sqrt $(\exp(A)-\sin(A))/x\uparrow 2$

53. Write down the effect of the following **for** statements.

i) **for** $i:=1$ **step** 1 **until** 10 **do** S; ii) **for** $i:=-4$ **step** 2 **until** 7 **do** S;

iii) **for** $x:=0$ **step** $0\cdot 1$ **until** 1 **do** S; iv) **for** $x:=1$ **step** $-0\cdot 1$ **until** $-0\cdot 5$ **do** S;

v) **for** $x:=5$ **step** 1 **until** 4 **do** S;

54. Write the effect of executing the assignment statements of the following ALGOL block.

```
begin real a,b,c;
        c:= 5;
        a := 4·1;
        b:= 2×a+7;
        c:= 3×a−b;
end
```

55. Write the ALGOL program which generates an array K with $K[i]=i!$ for $i=1,2,\ldots\ldots,10$.

56. Write ALGOL program to compute the mean of 10 observations.

57. Write an ALGOL program for finding the area of a triangle, given its three sides.

58. Two one-dimensional arrays X and Y each contain 50 elements. Write the ALGOL program to compute

$LX = \sqrt{\sum_{j=1}^{50} X_j^2}$ (Length of the vector X), $LY = \sqrt{\sum_{j=1}^{50} Y_j^2}$ (Length of the vector Y)

INPROD $= \sum_{j=1}^{50} X_j Y_j$ (Inner product of X and Y)

59. Write ALGOL program to multiply the matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ by a column matrix $B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

60. Write down the ALGOL block for finding the roots of $ax^2 + bx + c = 0, (a \neq 0)$.

UNIT - 4 (BOOLEAN ALGEBRAS)

61. What is two element Boolean algebra.

62. In a Boolean algebra show that $x \leq z \Rightarrow x \vee (y \wedge z) = (x \vee y) \wedge z$.

63. In a Boolean algebra prove that $(x \wedge y') \vee (x' \wedge y) = (x \vee y) \wedge (x' \vee y')$.

64. In a Boolean algebra prove that $(x \wedge y) \vee (y \wedge z) \vee (z \wedge x) = (x \vee y) \wedge (y \vee z) \wedge (z \vee x)$.

65. Let $a, b \in B$, a Boolean algebra. If $\vee$ is denoted by $+$ then prove that $a + b$ is an upper bound for the set $\{a, b\}$ and also $a + b = \sup\{a, b\}$.

66. Given the interval $[a, b]$ of a Boolean algebra A. Show that the algebraic system $[[a,b], \wedge, \vee, *, a.b]$ is a Boolean algebra, where $x^* = (a \vee x') \wedge b, \forall x \in [a, b]$.

67. Draw the block diagram of $p \wedge (p \vee q)$.

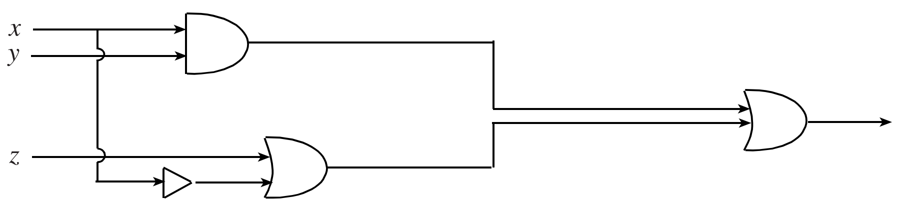
68. Draw the block diagram of $(A_1 \wedge A_2) \vee (A_1' \wedge A_2')$

69. Draw the block diagram of $ABC \vee A'B'C \vee A'B'C'$.

70. Write the gating network representing the Boolean expression $(x \vee y) \wedge (x' \vee y' \vee z') \wedge (y' \vee z)$.

71. Write the gating network representing the Boolean expression $[(x_1 \vee x_2) \wedge x_3'] \vee (x_1 \wedge x_2)$.

72. Write the Boolean expression for the gating network.



73. Show that $[(p \Rightarrow q) \wedge (p \Rightarrow r)] \Rightarrow (p \Rightarrow r)$ is a tautology.

74. Show that $(p \rightarrow q) \rightarrow [(r \vee p) \rightarrow (r \vee q)]$ is tautology, regardless of r.

75. Show that $(p' \to p) \wedge (p \to p')$ is an absurdity.

76. Construct the truth table and write the logic diagram for the following Boolean polynomial, $P(x, y, z) = (x \wedge y) \vee (y \wedge z')$.

77. Construct the truth table and write the logic diagram for the following Boolean polynomial, $P(x, y, z) = (x \wedge z) \vee (x' \wedge y) \vee (y \wedge z)$

78. Write an ALGOL program to compute $F(x) = \begin{cases} 17\cdot 3 - (x+1)^x & x < 5 \\ 19\cdot 4/(1+x^2) & x \geq 5 \end{cases}$ for x ranging from 0 to 10 in steps of $0\cdot 1$.

79. Write an ALGOL program to compute $F(x) = \begin{cases} 3125 - x^x & \text{if } x < 5 \\ (x-5)/(1+x^2) & \text{if } x \geq 5 \end{cases}$ for x ranging from 1 to 10 in steps of $0\cdot 1$.

80. Write the ALGOL program which computes the relation matrix for the relation ρ^2 where ρ is a binary relation on a set $X = \{x_1, x_2, \ldots\ldots, x_n\}$.

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ACHARYA NAGARJUNA UNIVERSITY

B.A / B.Sc. DEGREE EXAMINATION, THEORY MODEL PAPER

(Examination at the end of third year, for 2010 - 2011 and onwards)

MATHEMATICS PAPER - IV (ELECTIVE - 2)

MODERN APPLIED ALGEBRA

Time : 3 Hours **Max. Marks : 100**

SECTION - A (6 x 6 = 36 Marks)

Answer any ***SIX*** questions. Each question carries 6 marks

1. State Peano axioms.

2. Show that the relation $m \leqq n$ means $m|n$ (meaning that *m* is a divisor of *n*) is a partial ordering of the set of all positive integers.

3. Explain utilities problem.

4. If a graph (connected or disconnected) has exactly two vertices of odd degree, prove that there must be a path joining these two vertices.

5. Draw the state diagram for the following machine.

Present state	v		ξ	
	0	1	0	1
1	1	2	0	0
2	2	3	0	0
3	3	4	0	0
4	4	1	0	1

6. Write ALGOL expressions for i) $\dfrac{-b+\sqrt{b^2-4ac}}{2a}$ ii) $\sin^3\left(\dfrac{x}{y}\right)$ iii) $\dfrac{a^b+c^d}{4^q}$

7. Define Boolean algebra.

8. Prove that in any Boolean algebra, $a \wedge x = 0$ and $a \vee x = 1$ imply $x = a'$.

SECTION - B (4 x 16 = 64 Marks)

Answer ***ALL*** questions. Each question carries 16 marks

9.(a) Prove that a function is left invertible iff it is one one.

(b) Prove by induction that $\sum_{k=1}^{n} k^2 = \dfrac{n(n+1)(2n+1)}{6}$ where *n* is any positive integer.

OR

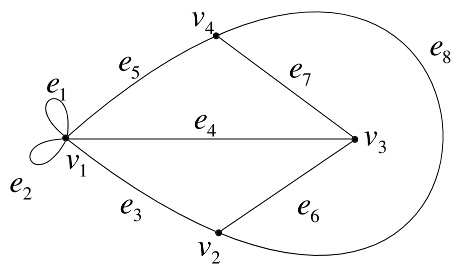
10.(a) Prove that an equivalence relation on a set *S* gives rise to a partition on *S*.

(b) If ρ and σ are reflexive and symmetric relations on a set *S*, then show that the following are equivalent.

i) $\rho\sigma$ is symmetric ii) $\rho\sigma = \sigma\rho$ iii) $\rho\sigma = \sigma \vee \rho$.

11.(a) Prove that a connected graph *G* is an Euler graph iff it can be decomposed into circuits.

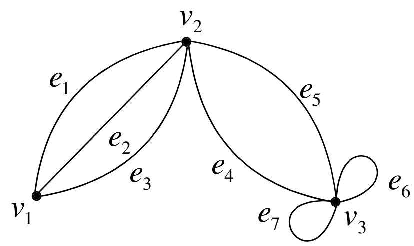
(b) Find the ajacency matrix and incidence matrix of the graph given by



OR

12.(a) Prove that the number of vertices of odd degree in a graph is always even.

(b) Draw all the circuits of the following graph.



13.(a) Prove that the relation of equivalence of machines is an equivalence relation.

(b) Minimize the number of states in the following machine.

Present state	Next state		Output	
	a_0	a_1	a_0	a_1
1	9	1	1	1
2	2	2	1	0
3	7	5	0	1
4	2	2	1	0
5	2	2	1	0
6	3	9	1	0
7	6	8	1	0
8	9	9	1	0
9	4	6	0	0

OR

14.(a) Explain i) **for** statement ii) **BLOCK** structures in ALGOL.

(b) Two one-dimensional arrays X and Y each contain 50 elements. Write the ALGOL program to compute $LX = \sqrt{\sum_{j=1}^{50} X_j^{\,2}}$ (Length of the vector X), $LY = \sqrt{\sum_{j=1}^{50} Y_j^{\,2}}$ (Length of the vector Y),

INPROD $= \sum_{j=1}^{50} X_j Y_j$ (Inner product of X and Y)

15.(a) In any Boolean algebra, if $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$, then prove that $x = y$.

(b) Write the gating network representing the boolean expression $\left[(x_1 \vee x_2) \wedge x_3'\right] \vee (x_1 \wedge x_2)$.

OR

16.(a) If any Boolean algebra $\mathscr{B} = [A, \wedge, \vee, ']$, prove that the relation $a \leqq b$ is a partial ordering of A. Moreover, in terms of this partial ordering, prove that $a \wedge b = glb \{a, b\}$ and $a \vee b =$ lub $\{a, b\}$.

(b) Show that $(p \to q) \to [(r \vee p) \to (r \vee q)]$ is a tautology , regardless of r.

AACHARYA NAGARJUNA UNIVERSITY

B.A / B.Sc. DEGREE EXAMINATION, PRACTICAL MODEL PAPER

(Practical examination at the end of third year, for 2010 - 2011 and onwards)

MATHEMATICS PAPER - IV (ELECTIVE - 2)

MODERN APPLIED ALGEBRA

Time : 3 Hours **Max. Marks : 30**

Answer ***ALL*** questions. Each question carries $7\frac{1}{2}$ marks. $4\times 7\frac{1}{2}=30$ M

1 (a) If $f_1, f_2, \ldots\ldots, f_m$ are injections then show that $f_m \circ f_{m-1} \circ \ldots \circ f_1$ is an injection.

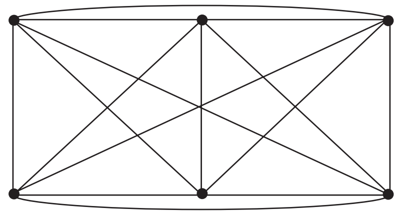
OR

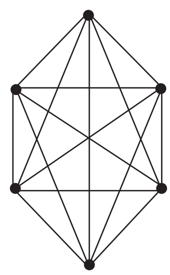
(b) If ρ is reflexive and transitive then show that $\rho \wedge \overline{\rho}$ is an equivalence relation on a set S.

2 (a) Draw a graph with the adjacency matrix $\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

OR

(b) Show that the two graphs are isomorphic.





3 (a) Give the state diagram of the following transition table.

Present state	Next state		Output	
	0	1	0	1
s_1	s_3	s_2	0	0
s_2	s_1	s_4	1	0
s_3	s_2	s_1	1	1
s_4	s_4	s_3	1	0

OR

(b) Convert the following ALGOL expressions to conventional mathematical expressions.

i) $A\times B-C$ ii) $A\uparrow(B-C)$ iii) $A\uparrow B-C$ iv) $A/B-C\uparrow D$ v) $A\div B-C\times D$

Evaluate the above for $A=2, B=3, C=4, D=5$

4 (a) Draw the block diagram of $ABC\vee A'B'C\vee A'B'C'$.

OR

(b) Show that $(p'\to p)\wedge(p\to p')$ is an absurdity.

Written exam	**: 30 Marks**
For record	**: 10 Marks**
For viva-voce	**: 10 Marks**
Total marks	**: 50 Marks**