ACHARYA NAGARJUNA UNIVERSITY

B.A / B.Sc. MATHEMATICS - PAPER - I

DIFFERENTIAL EQUATIONS AND SOLID GEOMETRY

PROBLEMS FOR PRACTICALS

UNIT - 1

(Differential equations of first order and first degree, not of the first degree)

1. Solve $(1-x^2)\frac{dy}{dx}+2xy=x(1-x^2)^{1/2}$.
2. Solve $(x+2y^3)\frac{dy}{dx}=y$.
3. Solve $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$.
4. Solve $x^2y\,dx-(x^3+y^3)dy=0$.
5. Solve $(x^2y-2xy^2)dx-(x^3-3x^2y)dy=0$.
6. Solve $(xy^2+2x^2y^3)dx+(x^2y-x^3y^2)dy=0$.
7. Solve $(x^2y^2+xy+1)y\,dx+(x^2y^2-xy+1)x\,dy=0$.
8. Solve $\left(y+\frac{1}{3}y^3+\frac{1}{2}x^2\right)dx+\frac{1}{4}(x+xy^2)dy=0$.
9. Solve $(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0$.
10. Solve $(2y\,dx+3x\,dy)+2xy(3y\,dx+4x\,dy)=0$.
11. Solve $\sec^2 y\frac{dy}{dx}+2x\tan y=x^3$.
12. Solve $\frac{dx}{y-z}=\frac{dy}{z-x}=\frac{dz}{x-y}$.
13. Solve $\frac{dx}{x(y^2-z^2)}=\frac{dy}{y(z^2-x^2)}=\frac{dz}{z(x^2-y^2)}$.
14. Show that the system of confocal and coaxial parabolas $y^2=4a(x+a)$ is self orthogonal.
15. Find the orthogonal trajectories of the family of cardioids $r=a(1-\cos\theta)$ where a is the parameter.
16. Solve $p^2+2py\cot x=y^2$.
17. Solve $y+px=p^2x^4$.
18. Solve $y^2\log y=xyp+p^2$.
19. Solve $x^2(y-px)=yp^2$.
20. Solve $(px-y)(py+x)=2p$.

UNIT - 2

(Higher order linear differential equations & system of linear differential equations)

21. Solve $(D^3-D^2-4D+4)y=e^{3x}$.
22. Solve $(D^2-3D+2)y=\cosh x$.

23. Solve $(D^2-4D+3)y=\sin 3x\cos 2x$.
24. Solve $(D^3+1)y=\cos 2x$.
25. Solve $(D^3+3D^2+2D)y=x^2$.
26. Solve $(D^2+3D+2)y=e^{-x}+x^2+\cos x$.
27. Solve $(D^2-2D+5)y=e^{2x}\sin x$
28. Solve $(D^4-1)y=e^x\cos x$.
29. Solve $(D^3-3D^2+3D-1)y=(x+1)e^x$.
30. Solve $(D^2-2D+1)y=xe^x\sin x$.
31. Solve $x^2\frac{d^2y}{dx^2}-(x^2+2x)\frac{dy}{dx}+(x+2)y=x^3e^x$.
32. Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x$ given that $y=x$ is a solution of $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$.
33. Solve $(D^2+1)y=\operatorname{cosec} x$ by variation of parameters.
34. Solve $(D^2+4D+4)y=3xe^{-x}$ by variation of parameters.
35. Solve $(x^2D^2-xD-3)y=x^2\log x$.
36. Solve $(x^2D^2+3xD+1)y=\frac{1}{(1-x)^2}$.
37. Solve $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=2\sin\ \log(1+x)$.
38. Solve $2\frac{dx}{dt}-x+\frac{dy}{dt}+4y=1,\frac{dx}{dt}-\frac{dy}{dt}=t-1$.
39. Solve the system $(D+4)x+Dy=1,(D-2)x+y=t^2$.
40. Solve $\frac{dy}{dt}=x-2y,\frac{dy}{dt}=5x+3y$.

UNIT - 3

(The plane, the line and the spheres)

41. Find the equation of the plane passing through the points $(2,2,-1),(3,4,2),(7,0,6)$.
42. A variable plane is at a constant distance $3p$ from the origin and meets the axes in A,B and C. Show that the locus of the centroid of the triangle ABC is $x^{-2}+y^{-2}+z^{-2}=p^{-2}$.
43. Find the equation of the plane through the line of inersection of the planes $2x+3y+4z-7=0$, $x+y+z-1=0$ and perpendicular to the plane $x-5y+3z-2=0$.
44. Find the bisecting plane of acute angle between the planes $3x-2y+6z+8=0$, $2x-y+2z+3=0$.
45. Find the equation of the plane that bisects the angle between the planes $3x-6y+2z+5=0$, $4x-12y+3z-3=0$ and which contains the origin.
46. Show that $x^2+4y^2+9z^2-12yz-6zx+4xy+5x+10y-15z+6=0$ represents a pair of parallel planes and find the distance between them.

47. Find the equation of the line through the point $(1,2,4)$ and parallel to the line $3x+2y-z=4$, $x-2y-2z=5$.

48. Show that the lines $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ and $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ are coplanar. Find their point of intersection and the plane containing the lines.

49. Show that the lines $\frac{x+5}{3}=\frac{y+4}{1}=\frac{z-7}{-2}$ and $3x+2y+z-2=0=x-3y+2z-3$ are coplanar and find the equation to the plane containing them.

50. Show that the lines $7x-4y+7z+16=0=4x+3y-2z+3$ and $x-3y+4z+6=0$ $=x-y+z+1$ are coplanar and find the plane containing the lines.

51. Find the shortest distance and the equations of S.D. line between the lines $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$; $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$.

52. Find the shortest distance and the equations of S.D. line between the lines $\frac{x}{4}=\frac{y+1}{3}=\frac{z-2}{2}$; $5x-2y-3z+6=0=x-3y+2z-3$.

53. Find the length of the perpendicular form $(4,-5,3)$ to the line $\frac{x-5}{3}=\frac{y+2}{-4}=\frac{z-6}{5}$.

54. Find the equation of the sphere through the points $(4,-1,2), (0,-2,3), (1,5,-1), (2,0,1)$.

55. Find the centre and radius of the circle $x^2+y^2+z^2-2y-4z=11,\ x+2y+2z=15$.

56. Find the equation of the sphere through the circle $x^2+y^2+z^2=9, 2x+3y+4z=5$ and the point $(1,2,3)$.

57. Show that the two circles $x^2+y^2+z^2-y+2z=0,\ x-y+z-2=0$;

$x^2+y^2+z^2+x-3y+z-5=0,\ 2x-y+4z-1=0$ lie on the same sphere and find its equation.

58. Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the point of contact.

59. Find the equation of the sphere which touches the plane $3x+2y-z+2=0$ at $(1,-2,1)$ and cuts orthogonally the sphere $x^2+y^2+z^2-4x+6y+4=0$.

60. Find the limiting points of the coaxal system defind by the spheres $x^2+y^2+z^2+3x-3y+6=0$, $x^2+y^2+z^2-6y-6z+6=0$.

UNIT - 4

(Cones, Cylinders and Conicoids)

61. Find the equation of the cone whose vertex is $(1,1,0)$ and whose guiding curve is $y=0, x^2+z^2=4$.

62. Find the enveloping cone of the sphere $x^2+y^2+z^2-2x+4z=1$ with its vertex at $(1,1,1)$.

63. Find the equation to the cone which passes through the three coordinate axes and the lines $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and $\frac{x}{3}=\frac{y}{-1}=\frac{z}{1}$.

64. Planes through OX and OY include an angle α. Show that their line of intersection lies on the cone $z^2(x^2+y^2+z^2)=x^2y^2\tan^2\alpha$.

65. Find the angle between the lines of intersection of the plane $x-3y+z=0$ and the cone $x^2-5y^2+z^2=0$.
66. Show that the plane $ax+by+cz=0$ cuts the cone $yz+zx+xy=0$ in perpendicular lines if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$.
67. Show that the equation $x^2-2y^2+3z^2-4xy+5yz-6zx+8x-19y-2z-20=0$ represents a cone with vertex $(1,-2,3)$.
68. If $\frac{x}{1}=\frac{y}{2}=\frac{z}{1}$ represents one of a set of three mutually perpendicular generators of the cone $11yz+6zx-14xy=0$, find the equations of the other two.
69. Show that the general equation of a cone which touches the coordinate planes is $\sqrt{fx}\pm\sqrt{gy}\pm\sqrt{hz}=0, f, g, h$ being parameters.
70. Show that the tangent planes to the cone $x^2-y^2+2z^2-3yz+4zx-5xy=0$ are perpendicular to the generators of the cone $17x^2+8y^2+29z^2+28yz-46zx-16xy=0$.
71. Prove that the cones $ayz+bzx+cxy=0, (ax)^{1/2}+(by)^{1/2}+(cz)^{1/2}=0$ are reciprocal.
72. Find the equation of the right circular cone with its vertex at the origin, axis along the line $\frac{x}{2}=\frac{y}{-4}=\frac{z}{3}$ and passing through the point $(1,1,2)$.
73. Find the equation of the right circular cone generated by straight lines drawn from the origin to cut the circle through the points $(1,2,2),(2,1,-2)$ and $(2,-2,1)$.
74. Find the equation of the cylinder whose generators are parallel to $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and which passes through the curve $x^2+2y^2=1,\ z=3$.
75. Find the equation to the right circular cylinder whose guiding circle is $x^2+y^2+z^2=9,\ x-y+z=3$.
76. Find the equation to the right circular cylinder whose axis is $\frac{x-1}{2}=\frac{y}{3}=\frac{z-3}{1}$ and radius 2.
77. Find the equation of the enveloping cylinder of the sphere $x^2+y^2+z^2-2x+4y-1=0$ having its generators parallel to the line $x=y=z$.
78. Find the equation of the right circular cylinder whose guiding curve is the circle through the points $(1,0,0),(0,1,0),(0,0,1)$.
79. Find the points of intersection of the line $\frac{x+5}{-3}=\frac{y-4}{1}=\frac{z-11}{7}$ with the conicoid $12x^2-17y^2+7z^2=7$.
80. Show that the plane $3x+12y-6z-17=0$ touches the conicoid $3x^2-6y^2+9z^2+17=0$ and find the point of contact.