ACHARYA NAGARJUNA UNIVERSITY

CURRICULUM - B.A / B.Sc

MATHEMATICS - PAPER - IV (ELECTIVE - 1) 90 Hours

NUMERICAL ANALYSIS

(Syllabus for the academic years 2010-2011 and onwards)

UNIT - I **20 Hours**

Errors in Numerical computations : Numbers and their Accuracy, Errors and their Computation, Absolute, Relative and percentage errors, A general error formula, Error in a series approximation.

Solution of Algebraic and Transcendental Equations : The bisection method, The iteration method, The method of false position, Newton- Raphson method, Generalized Newton-Raphson method, Ramanujan's method, Muller's method.

UNIT - II **25 Hours**

Interpolation : Errors in polynomial interpolation, Forward differences, Backward differences, Central Differences, Symbolic relations, Detection of errors by use of D. Tables, Differences of a polynomial, Newton's formulae for interpolation formulae, Gauss's central difference formula, Stirling's central difference formula, Interpolation with unevenly spaced points, Lagrange's formula, Error in Lagrange's formula, Derivation of governing equations, End conditions, Divided differences and their properties, Newton's general interpolation.

UNIT - III **20 Hours**

Curve Fitting : Least-Squares curve fitting procedures, fitting a straight line, nonlinear curve fitting, Curve fitting by a sum of exponentials.

Numerical Differentiation and Numerical Integration : Numerical differentiation, Errors in numerical differentiation, Maximum and minimum values of a tabulated function, Numerical integration, Trapezoidal rule, Simpson's 1/3-rule, Simpson's 3/8 - rule, Boole's and Weddle's rule.

UNIT - IV **25 Hours**

Linear systems of equations : Solution of linear systems - Direct methods, Matrix inversion method, Gaussian elimination method, Method of factorization, Ill-conditioned linear systems. Iterative methods : Jacobi's method, Gauss-siedal method.

Numerical solution of ordinary differential equations : Introduction, Solution by Taylor's Series, Picard's method of Successive approximations, Euler's method, Modified Euler's method, Runge - Kutta methods, Predictor - Corrector methods, Milne's method.

Prescribed Text Book :- Scope as in Introductory methods of Numerical Analysis by S.S. Sastry, Prentice Hall India (4th edition), Chapter - 1 (1.2, 1.4, 1.5, 1.6); Chapter - 2 (2.2 - 2.7); Chapter - 3 (3.2, 3.3, 3.7.2, 3.9.1, 3.9.2, 3.10.1, 3.10.2); Chapter - 4 (4.2); Chapter - 5 (5.2 - 5.4.5); Chapter - 6 (6.3.2, 6.3.4, 6.3.7, 6.4); Chapter - 7 (7.2 - 7.5, 7.6.2)

Reference Books :-

1. G. Sankar Rao New Age International Publishers, New - Hyderabad.
2. Finite Differences and Numerical Analysis by H.C. Saxena, S. Chand and Company, New Delhi.

ACHARYA NAGARJUNA UNIVERSITY

CURRICULUM - B.A / B.Sc

MATHEMATICS - PAPER - IV (ELECTIVE - 1)

NUMERICAL ANALYSIS

QUESTION BANK FOR PRACTICALS

UNIT - I

1. i) Which of the following numbers has the greatest precision. a) $4 \cdot 3201$ b) $4 \cdot 32$ c) $4 \cdot 320106$

ii) How many digits are to be taken in computing $\sqrt{20}$ so that the error does not exceed $0 \cdot 01\%$.

2. i) Sum the numbers $0 \cdot 1532, 15 \cdot 45, 0 \cdot 000354, 305 \cdot 1, 8 \cdot 12, 143 \cdot 3, 0 \cdot 0212, 0 \cdot 643$ and $0 \cdot 1734$ where in each of which all the given digits are correct.

ii) If $u = 5xy^2 / z^3$ then find relative maximum error in u, given that $\Delta x = \Delta y = \Delta z = 0 \cdot 001$ and $x = y = z = 1$.

3. Find a real root of the equation $f(x) = x^3 - x - 1 = 0$ by bisection method.

4. Find a real root of the equation $x^3 - 6x - 4 = 0$ by bisection method.

5. Find a positive root of the equation $xe^x = 1$, which lies between 0 and 1 by bisection method.

6. Find the root of $\tan x + x = 0$ upto two decimal places, which lies between 2 and $2 \cdot 1$ by bisection method.

7. Find a real root of the equation $x \log_{10} x = 1 \cdot 2$ by bisection method.

8. Find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$ by the method of false position upto three places of decimals.

9. Find a real root of the equation $x^3 - x^2 - 2 = 0$ by Regula-Falsi method.

10. Find the root of the equation $xe^x = \cos x$ using the Regula Falsi method correct to three decimal places.

11. Find the root of $x^3 + x - 1 = 0$ by iteration method, give that root lies near 1.

12. Find a real root of the equation $\cos x = 3x - 1$ correct to three decimal places, using iteration method.

13. Find by the iteration method, the root near $3 \cdot 8$, of the equation $2x - \log_{10} x = 7$ correct to four decimal places.

14. Find the real root of the equation $x^2 - 5x + 2 = 0$ by Newton-Raphson's method.

15. Find by Newton's method, the root of the equation $e^x = 4x$ which is near to 2 correct to three places of decimals.

16. Using Newton-Raphson method, establish the iterative formula $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$ to calculate the square root of *N*. Hence find the square root of 8.

17. Using Newton-Raphson method, establish the iterative formula $x_{n+1} = \frac{1}{3}\left(2x_n + \frac{N}{x_n^2}\right)$ to calculate the cube root of *N*. Hence find the cube root of 12 applying the Newton-Raphson formula twice.

18. Find a double root of the equation $f(x) = x^3 - x^2 - x + 1 = 0$ by generalized Newton's method.

19. Find a root of the equation $xe^x = 1$ by Ramanujan's method.

20. Find the root of the equation $y(x) = x^3 - 2x - 5 = 0$, which lies between 2 and 3 by Muller's method.

21. Show that i) $(1+\Delta)(1-\nabla) = 1$ ii) $E\nabla = \Delta$ iii) $\delta = E^{-1/2}\Delta = \Delta E^{-1/2}$ iv) $\nabla - \Delta = \Delta\nabla = \delta^2$ v) $\mu^2 = 1 + \frac{1}{4}\delta^2$

22. Evaluate i) $\frac{\Delta^2 x^3}{Ex^3}$ ii) $\left(\frac{\Delta^2}{E}\right)x^3$, the interval of differencing being unity.

23. Prove that i) $u_3 = u_2 + \Delta u_1 + \Delta^2 u_0 + \Delta^3 u_0$ ii) $u_4 = u_3 + \Delta u_2 + \Delta^2 u_1 + \Delta^3 u_1$

24. Find the missing term in the following data.

x:	0	1	2	3	4
y:	1	3	9	?	81

25. Form a table of differences for the function $f(x) = x^3 + 5x - 7$ for $x = -1, 0, 1, 2, 3, 4, 5$ and continue the table to obtain $f(6)$ and $f(7)$.

26. Find the function whose first difference is $x^3 + 3x^2 + 5x + 12$, if 1 be the interval of differencing.

27. The population of a country in the decennial census were as under. Estimate the population for the year 1895.

Year (x):	1891	1901	1911	1921	1931
Population (y) (in thousands):	46	66	81	93	101

28. From the following find y value at $x = 0{\cdot}26$.

x	$0{\cdot}10$	$0{\cdot}15$	$0{\cdot}20$	$0{\cdot}25$	$0{\cdot}30$
$y = Tan\,x$	$0{\cdot}1003$	$0{\cdot}1511$	$0{\cdot}2027$	$0{\cdot}2553$	$0{\cdot}3093$

29. From the following table, find the number of students who obtain less than 56 marks.

Marks :	30-40	40-50	50-60	60-70	70-80
No. of students :	31	42	51	35	31

30. Find the cubic polynomial which takes the following values.

x :	0	1	2	3
$f(x)$:	0	2	1	10

31. If l_x represents the number of persons living at age x in a life table, find as accurately as the data will permit the value of l_{47}. Given that $l_{20} = 512, l_{30} = 439, l_{40} = 346, l_{50} = 243$.

32. Apply Gauss forward formula to find the value of u_9 if $u_0 = 14; u_4 = 24; u_8 = 32; u_{16} = 40$.

33. Given that $\sqrt{12500} = 111 \cdot 803399$; $\sqrt{12510} = 111 \cdot 848111$; $\sqrt{12520} = 111 \cdot 892806$;

$\sqrt{12530} = 111 \cdot 937483$. Show by Gauss backward formula that $\sqrt{12516} = 111 \cdot 874930$.

34. Use Stirling's formula to find y_{28}, given $y_{20} = 49225, y_{25} = 48316, y_{30} = 47236, y_{35} = 45926$, $y_{40} = 44306$.

35. Given $y_{20} = 24, y_{24} = 32, y_{28} = 35, y_{32} = 40$, find y_{25} by Bessel's formula.

36. By means of Newton's divided difference formula, find the value $f(8)$ and $f(15)$ from the following table :

x :	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

37. Using the Newton's divided difference formula, find a polynomial function satisfying the following data.

x :	-4	-1	0	2	5
$f(x)$:	1245	33	5	9	1335

38. Using Lagranges interpolation formula find y at $x = 301$.

x:	300	304	305	307
y:	$2 \cdot 4771$	$2 \cdot 4829$	$2 \cdot 4843$	$2 \cdot 4871$

39. By Lagrange's interpolation formula, find the form of the function given by

x :	0	1	2	3	4
$f(x)$:	3	6	11	18	27

40. Using Lagrange's formula, prove that $y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8}\left[\frac{1}{2}(y_3 - y_1) - \frac{1}{2}(y_{-1} - y_{-3})\right]$.

41. Find the least square line for the data points $(-1,10), (0,9), (1,7), (2,5), (3,4), (4,3), (5,0)$ and $(6,-1)$.

42. Find the least square power function of the form $y = ax^b$ for the data.

x_i	1	2	3	4
y_i	3	12	21	35

43. Fit a second degree parabola to the following data :

x:	0	1	2	3	4
y:	1	$1 \cdot 8$	$1 \cdot 3$	$2 \cdot 5$	$6 \cdot 3$

44. Using the given table, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1\cdot 2$.

x	$1\cdot 0$	$1\cdot 2$	$1\cdot 4$	$1\cdot 6$	$1\cdot 8$	$2\cdot 0$	$2\cdot 2$
y	$2\cdot 7183$	$3\cdot 3201$	$4\cdot 0552$	$4\cdot 9530$	$6\cdot 0496$	$7\cdot 3891$	$9\cdot 0250$

45. From the following table, find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 2\cdot 03$

x	$1\cdot 96$	$1\cdot 98$	$2\cdot 00$	$2\cdot 02$	$2\cdot 04$
y	$0\cdot 7825$	$0\cdot 7739$	$0\cdot 7651$	$0\cdot 7563$	$0\cdot 7473$

46. Find $f'(0\cdot 6)$ and $f''(0\cdot 6)$ from the following table :

x	$0\cdot 4$	$0\cdot 5$	$0\cdot 6$	$0\cdot 7$	$0\cdot 8$
$f(x)$	$1\cdot 5836$	$1\cdot 7974$	$2\cdot 0442$	$2\cdot 3275$	$2\cdot 6510$

47. Find $f'(2\cdot 5)$ from the following table :

x	$1\cdot 5$	$1\cdot 9$	$2\cdot 5$	$3\cdot 2$	$4\cdot 3$	$5\cdot 9$
$f(x)$	$3\cdot 375$	$6\cdot 059$	$13\cdot 625$	$29\cdot 368$	$73\cdot 907$	$196\cdot 579$

48. Find the maximum value of y, by using data given below :

x	0	1	2	3	4
y	0	$0\cdot 25$	0	$2\cdot 25$	16

49. Find $f'(1\cdot 5)$ and $f''(1\cdot 5)$ from the following table.

x	$1\cdot 5$	$2\cdot 0$	$2\cdot 5$	$3\cdot 0$	$3\cdot 5$	$4\cdot 0$
$f(x)$	$3\cdot 375$	$7\cdot 000$	$13\cdot 625$	$24\cdot 000$	$38\cdot 875$	$59\cdot 000$

50. Assuming Stirling's formula, show that $\frac{d}{dx}[f(x)] = \frac{2}{3}[f(x+1) - f(x-1)] - \frac{1}{12}[f(x+2) - f(x-2)]$ upto third differences.

51. Evaluate $I = \int_0^1 \frac{dx}{1+x}$ correct to three decimal places by Trapezoidal rule with $h = 0\cdot 25$.

52. Evaluate $\int_0^1 (4x - 3x^2)\, dx$ taking 10 intervals by Trapezoidal rule.

53. Calculate an approximate value of $\int_0^{\pi/2} \sin x dx$ by Trapezoidal rule.

54. By Simpson's $\frac{1}{3}$ rule, evaluate $\int_1^2 \sqrt{1 - 1/x}\, dx$ with five ordinates.

55. Use Simpson's $\frac{1}{3}$ rule to prove that $\log_e 7$ is approximately $1\cdot9587$ using $\int_1^7 \frac{dx}{x}$.

56. Find the value of the integral $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule. Hence obtain the approximate value of π in each case.

57. Evaluate $\int_0^1 \frac{1}{1+x} dx$ by Boole's rule.

58. Evaluate the integral $\int_0^4 e^x \, dx$ by Boole's Rule.

59. Evaluate the integral $\int_4^{5\cdot2} \log x \, dx$, using Weddle's rule.

60. Integrate numerically $\int_0^{\pi/2} \sqrt{\sin x} \, dx$ by Weddle's rule.

61. Solve the equations $3x+2y+4z=7; 2x+y+z=7; x+3y+5z=2$ by matrix inversion method.

62. Solve the equations $x+y+z=9,\ 2x+5y+7z=52, 2x+y-z=0$ by Cramer's rule.

63. Solve the equations $2x-y+4z=12; 8x-3y+2z=23; 4x+11y-z=33$ by Gauss elimination method.

64. Solve the system of equations $2x+y+z=10, 3x+2y+3z=18, x+4y+9z=16$ by Gauss elimination method.

65. Solve the equations $3x+2y+4z=7; 2x+y+z=7; x+3y+5z=2$ by Factorization method.

66. Solve the equations $2x+3y+z=9, x+2y+3z=6, 3x+y+2z=8$ by LU decomposition method.

67. Solve the following equations by Gauss-Jacobi method : $10x-y+z=12;\ x-10y+z=12;$ $x+y-10z=12$ correct to 3 decimals.

68. Solve the following equations by Jacobi method : $20x+y-2z=17;\ 3x+20y-z=-18;$ $2x-3y+20z=25$.

69. By Gauss-seidel iterative method solve the linear equations $x_1+10x_2+x_3=6;\ 10x_1+x_2+x_3=6$ and $x_1+x_2+10x_3=6$.

70. Solve the following system by Gauss-Seidel method : $10x+2y+z=9, 2z+20y-2z=-44,$ $-2x+3y+10z=22$.

71. Solve the differential equation $\frac{dy}{dx}=x+y$, with $y(0)=1, x\in[0, 1]$ by Taylor series expansion to obtain y for $x=0\cdot1$.

72. Solve the equation $y' = x + y^2$ with $y_0 = 1$ when $x = 0$.

73. Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with $y = 1$, when $x = 0$. Find approximately the value of y for $x = 0 \cdot 1$ by Picard's method.

74. Given $\frac{dy}{dx} = x^3 + y,\ y(0) = 1$, compute $y = (0 \cdot 2)$ by Euler's method taking $h = 0 \cdot 01$.

75. Using Euler's method, compute $y(0 \cdot 5)$ for differential equation $\frac{dy}{dx} = y^2 - x^2$, with $y = 1$ when $x = 0$.

76. Determine the value of y when $x = 0 \cdot 1$ given that $y(0) = 1$ and $y' = x^2 + y$.

77. Solve $\frac{dy}{dx} = -2xy^2$ with $y(0) = 1$ and $h = 0 \cdot 2$ on the interval [0, 1] using Runge-Kutta fourth order method.

78. Given $\frac{dy}{dx} = y - x$ with $y(0) = 2$, find $y(0 \cdot 1)$ and $y(0 \cdot 2)$ correct to four decimal places, using Runge-Kutta fourth order method.

79. Using Milne method obtain $y_{(4)}$ from the given table of tabulated values and $y' = y^2 - x^2$

x	0	$0 \cdot 1$	$0 \cdot 2$	$0 \cdot 3$
y	1	$1 \cdot 11$	$1 \cdot 25$	$1 \cdot 42$
f	1	$1 \cdot 22$	$1 \cdot 52$	$1 \cdot 92$

80. Solve $y' = 2e^x - y$ at $x = 0 \cdot 4$ and $x = 0 \cdot 5$ by Milne's method, given their values at the four points.

x	0	$0 \cdot 1$	$0 \cdot 2$	$0 \cdot 3$
y	2	$2 \cdot 010$	$2 \cdot 040$	$2 \cdot 090$

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ACHARYA NAGARJUNA UNIVERSITY

B.A / B.Sc. DEGREE EXAMINATION, THEORY MODEL PAPER

(Examination at the end of third year, for 2010 - 2011 and onwards)

MATHEMATICS PAPER - IV (ELECTIVE - 1)

NUMERICAL ANALYSIS

Time : 3 Hours **Max. Marks : 100**

SECTION - A (6 x 6 = 36 Marks)

Answer any ***SIX*** questions. Each question carries 6 marks

1. Describe a general error formula.

2. Using Newton-Raphson method, establish the iterative formula $x_{n+1} = \frac{1}{3}\left(2x_n + \frac{N}{x_n^2}\right)$ to calculate the cube root of N.

3. Given $u_0 = 3, u_1 = 12, u_2 = 81, u_3 = 200, u_4 = 100, u_5 = 8$; find $\Delta^5 u_0$.

4. Given $y_{20} = 24, y_{24} = 32, y_{28} = 35, y_{32} = 40,$ find y_{25} by Bessel's formula.

5. Fit a second degree parabola to the following data :

x:	0	1	2	3	4
y:	1	5	10	22	38

6. Evaluate $I = \int_0^1 \frac{dx}{1+x}$ correct to three decimal places by Trapezoidal rule with $h = 0\cdot 25$.

7. Solve the equations $5x_1 - x_2 - 2x_3 = 142; x_1 - 3x_2 - x_3 = -31;\ 2x_1 - x_2 - 3x_3 = 5$ by using Gauss's elimination method.

8. Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with $y = 1$, when $x = 0$. Find approximately the value of y for $x = 0\cdot 1$ by Picard's method.

SECTION - B (4 x 16 = 64 Marks)

Answer ***ALL*** questions. Each question carries 16 marks

9.(a) Find a positive root of the equation $xe^x = 1$, which lies between 0 and 1 by using bisection method.

(b) Find the smallest root of the equation $f(x) = x^3 - 6x^2 + 11x - 6 = 0$ by Ramanujan's method.

OR

10.(a) Find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$ by the method of false position upto three places of decimals.

(b) Find the root of the equation $2x = \cos x + 3$ correct to three decimal places by fixed point iteration method.

11.(a) State and prove Newton-Gregory formula for farword interpolation with equal intervals.

(b) Using the Newton's divided difference formula, find a polynomial function satisfying the following data :

x :	-4	-1	0	2	5
$f(x)$:	1245	33	5	9	1335

OR

12.(a) State and prove Lagrange's interpolation formula.

(b) Using Gauss forward formula find u_{32} from the given data : $u_{20} = 14 \cdot 035, u_{25} = 13 \cdot 674,$ $u_{30} = 13 \cdot 257, u_{35} = 12 \cdot 734, u_{40} = 12 \cdot 089, u_{45} = 11 \cdot 309$.

13.(a) Find $f'(1 \cdot 5)$ and $f''(1 \cdot 5)$ from the following table.

x	$1 \cdot 5$	$2 \cdot 0$	$2 \cdot 5$	$3 \cdot 0$	$3 \cdot 5$	$4 \cdot 0$
$f(x)$	$3 \cdot 375$	$7 \cdot 000$	$13 \cdot 625$	$24 \cdot 000$	$38 \cdot 875$	$59 \cdot 000$

(b) Find the value of the integral $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule. Hence obtain the approximate value of π in each case.

OR

14.(a) Find the maximum value of y, by using data given below :

x	0	1	2	3	4
y	0	$0 \cdot 25$	0	$2 \cdot 25$	16

(b) Evaluate the integral $\int_4^{5 \cdot 2} \log x \, dx$, using Weddle's rule.

15.(a) Solve the equations $3x + 2y + 4z = 7; 2x + y + z = 7; x + 3y + 5z = 2$ by Factorization method.

(b) Determine the value of y when $x = 0 \cdot 1$ given that $y(0) = 1$ and $y' = x^2 + y$.

OR

16.(a) Solve the following equations by Gauss-Jacobi method : $10x - y + z = 12;\ x - 10y + z = 12;$ $x + y - 10z = 12$ correct to 3 decimals.

(b) Given $\frac{dy}{dx} = y - x$ with $y(0) = 2$, find $y(0 \cdot 1)$ and $y(0 \cdot 2)$ correct to four decimal places using RK fourth order method.

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AACHARYA NAGARJUNA UNIVERSITY

B.A / B.Sc. DEGREE EXAMINATION, PRACTICAL MODEL PAPER

(Practical examination at the end of third year, for 2010 - 2011 and onwards)

MATHEMATICS PAPER - IV (ELECTIVE - 1)

NUMERICAL ANALYSIS

Time : 3 Hours **Max. Marks : 30**

Answer ***ALL*** questions. Each question carries $7\frac{1}{2}$ marks. $4\times7\frac{1}{2}=30$ M

1 (a) Find a positive root of the equation $xe^x=1$, which lies between 0 and 1 by bisection method.

OR

(b) Using Newton-Raphson method, establish the iterative formula $x_{n+1}=\frac{1}{2}\left(x_n+\frac{N}{x_n}\right)$ to calculate the square root of *N*. Hence find the square root of 8.

2 (a) Find the missing term in the following data.

x:	0	1	2	3	4
y:	1	3	9	?	81

OR

(b) If l_x represents the number of persons living at age x in a life table, find as accurately as the data will permit the value of l_{47}. Given that $l_{20}=512, l_{30}=439, l_{40}=346, l_{50}=243$.

3 (a) Find $f'(0\cdot6)$ and $f''(0\cdot6)$ from the following table :

x	$0\cdot4$	$0\cdot5$	$0\cdot6$	$0\cdot7$	$0\cdot8$
$f(x)$	$1\cdot5836$	$1\cdot7974$	$2\cdot0442$	$2\cdot3275$	$2\cdot6510$

OR

(b) Find the value of the integral $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule. Hence obtain the approximate value of π in each case.

4 (a) Solve the system of equations $2x+y+z=10, 3x+2y+3z=18, x+4y+9z=16$ by Gauss elimination method.

OR

(b) Given $\frac{dy}{dx}=\frac{y-x}{y+x}$ with $y=1$, when $x=0$. Find approximately the value of y for $x=0\cdot1$ by Picard's method.

Written exam	**: 30 Marks**
For record	**: 10 Marks**
For viva-voce	**: 10 Marks**
Total marks	**: 50 Marks**